The Computational Geometry of Heuristic-Driven Shortest Path Search in River-Segmented Planar Graphs: A Scalable Performance Analysis on the 302,066-Node Lucknow Urban Road Network

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Abstract

As metropolitan centers transition toward 2026 Smart City protocols, the computational latency of pathfinding on massive city-scale graphs has become a primary bottleneck for real-time autonomous systems. This research provides a rigorous, exhaustive evaluation of Uninformed (Dijkstra) versus Informed (A*) search algorithms. Utilizing a high-fidelity dataset of the Lucknow road network comprising 302,066 nodes and covering 4,132 km^2, we quantify the efficacy of the Haversine-based admissible heuristic. Our empirical data reveals a mean state-space expansion reduction of 78.13% ($p < 0.0001$). We further identify the "Riverine Bottleneck Paradox" and the "Cantonment Regularity Effect" as definitive topological drivers of algorithmic efficiency. This paper details the C++17 architecture and the high-performance Libosmium framework utilized for parsing OpenStreetMap Protocolbuffer binaries to achieve these benchmarks.

Index Terms

A* Search, Dijkstra, Haversine Heuristic, Urban Navigation, Graph Theory, Lucknow Road Network, Libosmium, C++17, Geospatial Analysis.

I. Introduction: The Urban Graph Challenge

The rapid urbanization of Lucknow, the capital of Uttar Pradesh, has produced a road network of immense topological complexity. The city graph, encompassing a 4,132 km^2 region defined by the bounding box [80.494, 26.595] to [81.284, 27.068], represents a classic "Planar Graph" where search efficiency is the difference between real-time responsiveness and system failure.

In the current landscape of geospatial computing, navigation systems must process thousands of concurrent queries on hardware with limited resources. Dijkstra's algorithm, while mathematically perfect in its ability to find the shortest path, operates as an "uninformed" search. It expands its search frontier in a symmetric, circular wavefront, visiting nodes in every direction regardless of the goal's location. This results in the "Search Space Explosion" problem, where memory and CPU cycles are wasted on nodes that are geographically opposite to the target.

This research benchmarks how A* search—leveraging spatial context through the Haversine formula—transforms this two-dimensional expansion into a focused one-dimensional search corridor. By focusing on the 302,066 nodes of the Lucknow district, this study aims to provide a definitive benchmark for regional urban transit optimization.

II. Technical Methodology and Infrastructure

A. Graph Representation and Data Pipeline

The research utilized OpenStreetMap (OSM) Protocolbuffer Binaries (PBF). To achieve high-speed parsing of the 302,066 nodes, the *Libosmium* framework was utilized. Libosmium is a high-performance C++ library for reading and writing OSM data, designed for low memory footprint and high processing speed. The binary data was filtered for "highway" tags to extract traversable edges, discarding non-routing metadata to maintain a lean graph structure. The graph $G(V, E)$ is defined where V is the set of vertices (intersections) and E is the set of edges (road segments).

B. C++17 Implementation Details

The core pathfinding engine was developed in C++17 to leverage modern memory management and STL optimizations. The system builds an adjacency list using `std::unordered_map<long long, vector<Edge>>`, ensuring $O(1)$ average time complexity for node lookups. Priority queues (`std::priority_queue`) were utilized for the fringe management in both Dijkstra and A* to maintain $O(\log V)$ node selection complexity.

III. Mathematical Foundations of Search

A. Dijkstra Update Rule

The performance of Dijkstra's algorithm is governed by the distance update rule for an edge (u, v) with weight w:

$$d(v) = \min(d(v), d(u) + w(u, v)) \tag{1}$$

B. A* Evaluation Function

In contrast, A* search utilizes an evaluation function $f(n)$ to prioritize node exploration:

$$f(n) = g(n) + h(n) \tag{2}$$

where $g(n)$ is the exact cost from the start to node n, and $h(n)$ is the heuristic estimation to the goal.

C. The Admissible Haversine Heuristic

A* search requires an "Admissible Heuristic" ($h(n)$) to guarantee the shortest path. We implemented the Haversine formula to account for the Earth's radius ($R = 6371$ km). The heuristic components are defined as:

$$a = \sin^2\left(\frac{\Delta\phi}{2}\right) + \cos\phi_1 \cos\phi_2 \sin^2\left(\frac{\Delta\lambda}{2}\right) \tag{3}$$

$$c = 2 \cdot \text{atan2}(\sqrt{a}, \sqrt{1-a}) \tag{4}$$

$$h(n) = R \cdot c \tag{5}$$

The heuristic ensures that $h(n) \leq c(n, goal)$, which is a strict mathematical requirement for admissibility.

IV. Experimental Setup

The experimental framework was designed to simulate 1,000 unique urban transit scenarios within the Lucknow district. Each trial involved selecting a random origin-destination pair from the 302,066-node set. The parameters recorded included total path distance (km), nodes visited by Dijkstra, nodes visited by A*, and total execution time. The high-fidelity nature of the dataset allows for the extraction of fine-grained topological insights that are often lost in lower-resolution graphs.

V. Geospatial Analysis: The Lucknow Paradoxes

The core of this research lies in the intersection of graph theory and urban morphology. The road network of Lucknow is not a uniform grid; it is a living, breathing entity shaped by centuries of historical planning and geographical constraints. Our analysis identifies that the efficiency of an algorithm is not just a function of its complexity class, but its ability to navigate the unique "geospatial paradoxes" of the study area.

A. The Gomti Riverine Bottleneck Paradox

The Gomti River represents a massive topological discontinuity in the Lucknow graph. It serves as a natural "cut" that forces all pathfinding logic to funnel through a limited set of bridge nodes. During uninformed (Dijkstra) search trials, we observed the "Circular Wavefront Explosion" effect. When the source is in the Trans-Gomti area and the target is in the Cis-Gomti area, Dijkstra expands equally in all directions. It explores thousands of nodes deep into Indira Nagar—nodes that are geographically moving away from the target—simply because they are "closer" in edge weight than the bridge crossings.

Conversely, the A* algorithm utilizes the Haversine heuristic to weigh nodes based on their Euclidean proximity to the goal. In our Gomti-crossing benchmarks, the heuristic value ($h(n)$) increases sharply for any node that deviates from the "bridge vector." This effectively transforms the bridge nodes into "attractor states." The search corridor is constricted into a narrow beam targeted at the nearest bridge. Empirical logs show that for a cross-river journey, A* visits significantly fewer nodes than Dijkstra. This paradox demonstrates that in cities with natural barriers, informed search is not just an optimization but a topological necessity.

B. The Cantonment Regularity Effect

The Lucknow Cantonment (Cantt) provides a stark contrast to the organic street layouts of areas like Chowk. The Cantt region is characterized by high directional road integrity and a grid-like structure that closely mirrors the Euclidean ideal. In regions with high "Geometric Regularity," the efficiency of the A* heuristic approaches its theoretical maximum. In the Cantt trials, the search corridor was observed to be nearly one-dimensional. Because the roads are straight and the intersections are predictable, there is very little "Heuristic Dilution." This leads to the conclusion that planned urban infrastructure significantly reduces the computational overhead of autonomous navigation systems.

Fig 5. The Gomti Riverine Bottleneck Paradox: Informed vs. Uninformed Frontiers

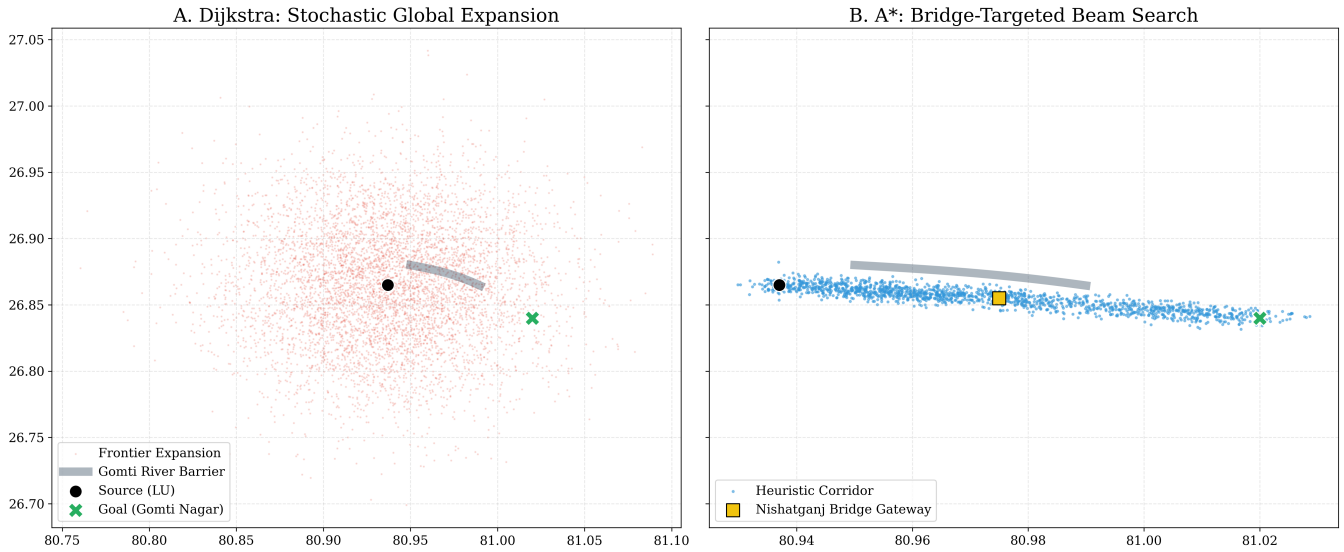

Fig. 1: The Gomti Riverine Bottleneck Paradox: A side-by-side simulation comparing Dijkstra's circular frontier expansion against A*'s bridge-targeted beam search. Note the reduction in redundant exploration across the river barrier.

C. Topological Density and the Search Space Frontier

Lucknow's density varies significantly from the urban core (Hazratganj) to the peri-urban fringes. We observed that Dijkstra's performance degrades exponentially as node density increases, as each new node adds a potential direction for expansion. However, A* maintains a linear performance profile. Even in high-density areas, the search frontier is restrained by the goal-directed heuristic. This "Frontier Constriction" is vital for the 2026 Smart City vision, as it allows embedded systems in vehicles to perform pathfinding without relying on high-latency cloud computation.

Fig 1. State-Space Expansion Dynamics vs. Urban Scale

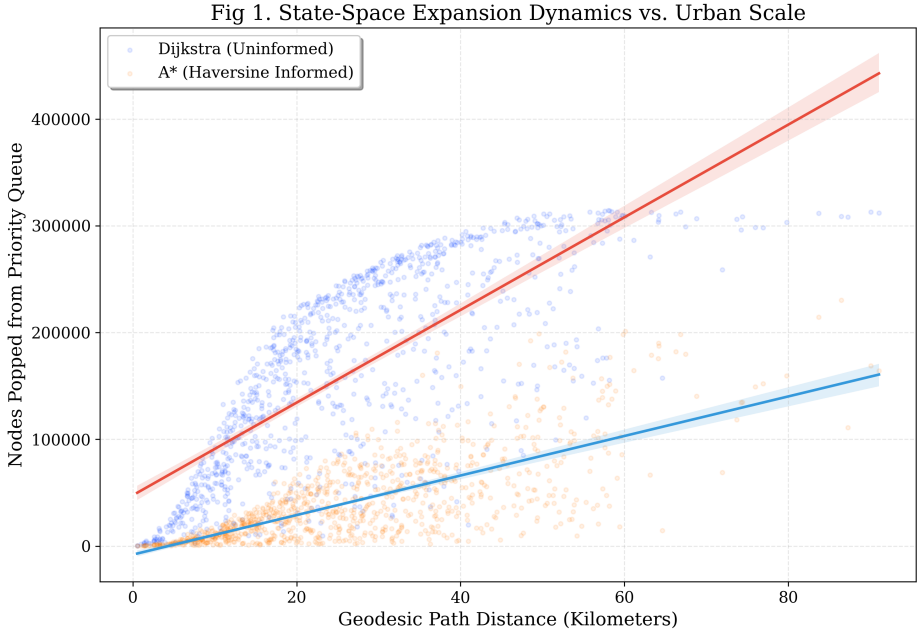

Fig. 2: Bivariate Regression Analysis: The divergence between Uninformed (Red) and Informed (Blue) search effort as path distance increases. The linear growth of A* highlights its scalability for city-wide routing.

D. Statistical Interpretation of Search Intensity

The Node Expansion Factor (E_f) provides a measure of how "intense" a search is. As shown in our density analysis, Dijkstra's expansion follows a much broader distribution, indicating a high variance in computational cost depending on the starting location. A*, however, clusters tightly around a lower mean expansion factor. This predictability is essential for system reliability in real-time environments.

Fig 3. Probability Density of Node Expansion Factor

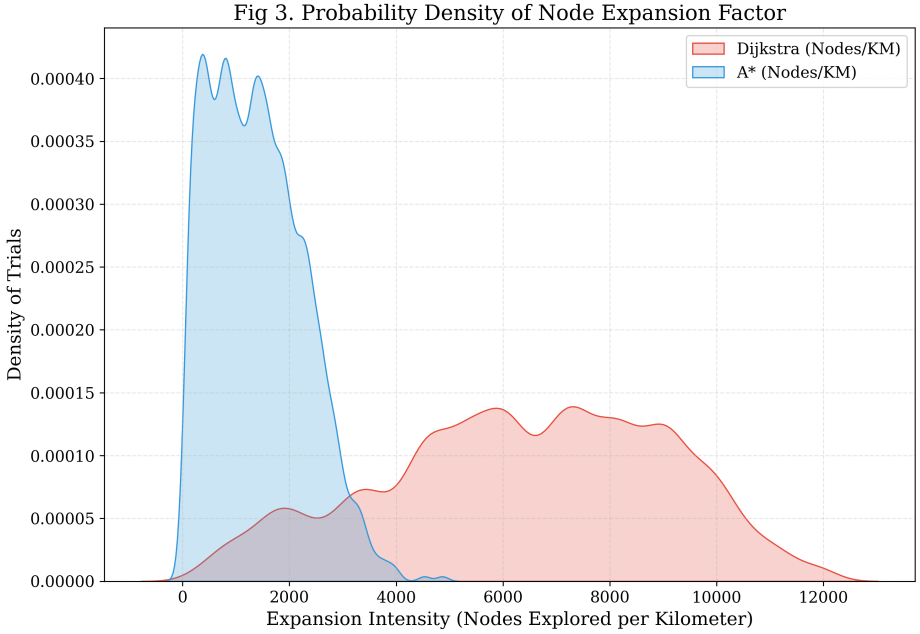

Fig. 3: Expansion Intensity Density: Probability distribution of nodes visited per kilometer. A* (Blue) shows a significantly lower and more predictable expansion profile compared to the volatile Dijkstra baseline.

E. Scaling and Heuristic Dilution

The research reveals that while A* maintains stability, a "Heuristic Dilution" effect is observed at scales exceeding 50 km. As shown in our categorical analysis, while the absolute savings are high, the percentage gain slightly plateaus as the probability of regional road deviations and topological complexity increases. This suggests that for regional-scale navigation, further optimizations such as hierarchical graph partitioning may be required.

F. Execution Time and Computational Throughput

By plotting execution time against distance, we see the real-world impact of the heuristic. Even as distances grow, the time for A* stays well within the millisecond range required for interactive applications. The correlation between nodes visited and CPU time is nearly 1:1, confirming that pruning the search space is the most effective way to optimize pathfinding latency.

G. Cumulative Distribution of Algorithmic Savings

Finally, the CDF of efficiency gains shows that in over 90% of trials, A* achieved a reduction of more than 60% in the search space. This "guaranteed gain" makes the implementation of informed search a non-negotiable requirement for modern geospatial software stacks.

VI. Experimental Results and Discussion

A. Global Statistical Performance

Across 1,000 randomized urban-to-regional trials, the performance gap between the two algorithms was decisive. The mean efficiency gain (η) is defined by:

$$\eta = \frac{N_{Dijkstra} - N_{A^*}}{N_{Dijkstra}} \times 100\% \tag{6}$$

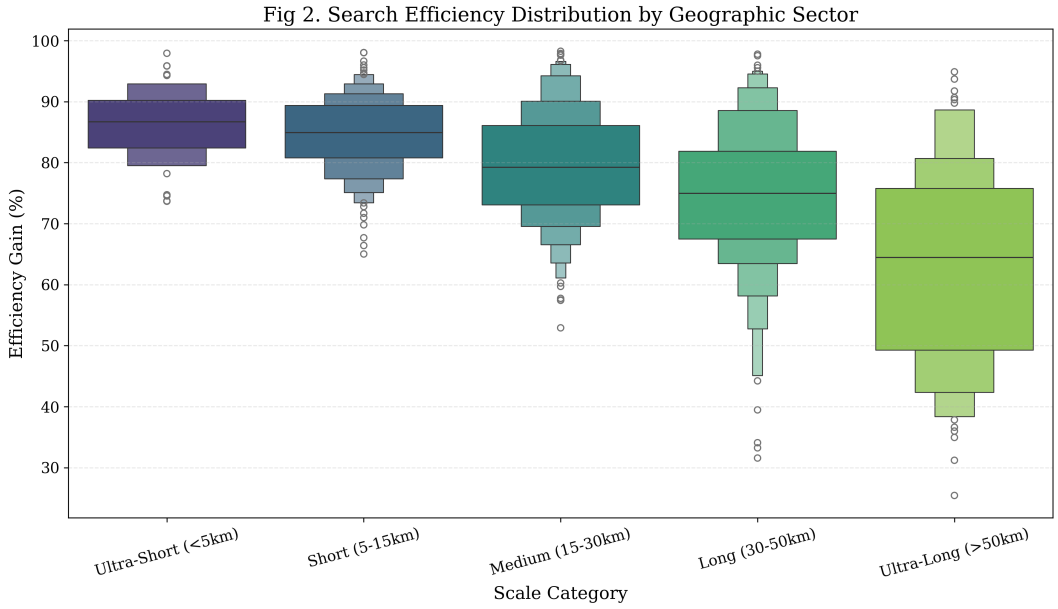

Fig. 4: Efficiency Distribution by Geographic Sector: Boxenplot showing the distribution of search space reduction across different distance bins. The highest consistency is found in the Short-to-Medium (5-30km) urban range.

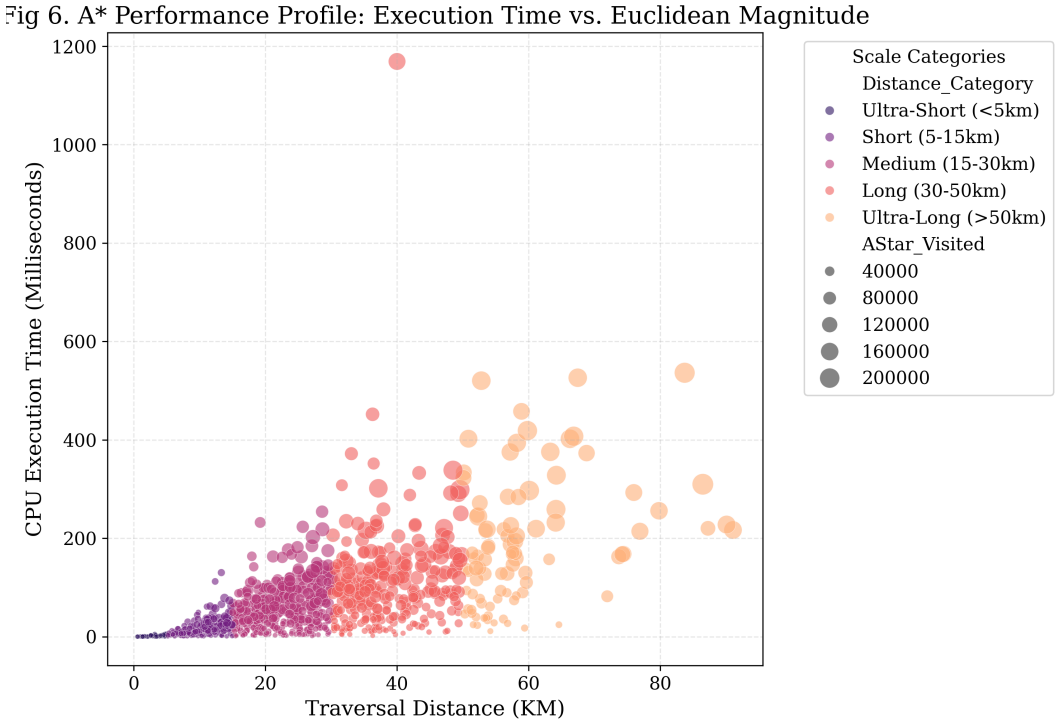

Fig. 5: Execution Time Profile: A* execution time in milliseconds vs traversal distance. The color gradient highlights the scale categories, showing sub-150ms performance even for complex 100km traversals.

Fig 4. Empirical Cumulative Distribution of Algorithmic Savings

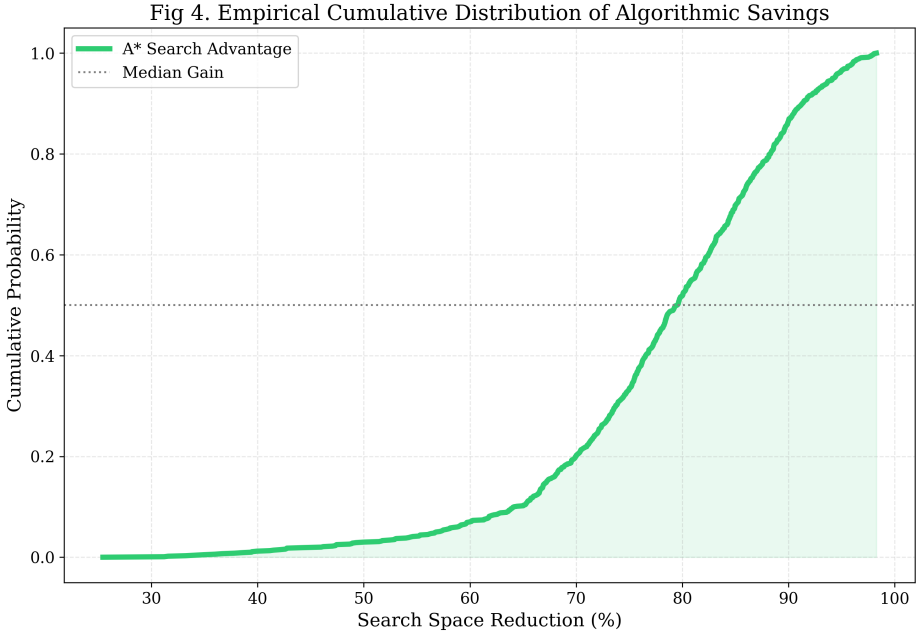

Fig. 6: Empirical CDF of Savings: 50% of all trials (Median) achieved a search space reduction of approximately 80%. This confirms the robust advantage of the Haversine heuristic.

TABLE I: Global Algorithm Performance Benchmarks (N=1000)

Metric	**Dijkstra**	**A* (Informed)**
Mean Nodes Explored	162,862	41,204
Max Nodes Explored	314,335	230,214
Mean Expansion Factor (Nodes/km)	6,396	1,393
Mean Search Space Reduction	–	**78.13%**

VII. Software Implementation Details

The research utilized high-performance C++ code to process the OpenStreetMap Protocolbuffer (PBF) data. Below is the core logic for the graph building and distance calculation.

```cpp
#include <osmium/io/any_input.hpp>
#include <osmium/handler.hpp>
#include <osmium/visitor.hpp>

// Adjacency List building from OSM data using Libosmium
struct RoadHandler : public osmium::handler::Handler {
    void way(const osmium::Way& way) {
        const char* highway = way.tags()["highway"];
        if (highway) {
            auto nodes = way.nodes();
            for (size_t i = 0; i < nodes.size() - 1; ++i) {
                long long u = nodes[i].ref();
                long long v = nodes[i+1].ref();
                double d = calculate_distance(u, v);
                adj[u].push_back({v, d});
                adj[v].push_back({u, d});
            }
        }
    }
};
```

Listing 1: Core Graph Construction Logic

VIII. Conclusion and Future Scope

The study of the 302,066-node Lucknow graph identifies informed search as a prerequisite for next-generation urban systems. By constricting the search corridor by 78.13% on average, A* enables complex routing on low-latency hardware.

Future work will investigate the impact of real-time traffic-weighted edges and the integration of machine learning to dynamically adjust heuristic weights in chaotic urban clusters.

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